**UNIC Mathematics Summer School, June 2024**

**Topic: Spectral Theory**
**Instructor: Prof. Dr. Luka Grubišić, Faculty of Science University of Zagreb**

1. **Course description**: Spectral Theory is an area of mathematics whose basic objects of study are self-adjoint operators on Hilbert spaces.

2. **Course goals:**
   - In this lecture series we first introduce a standard spectral theory of self-adjoint operators in a Hilbert space. As prototypes we will take operators defined by differential expressions in the spaces of square integrable funcions.
   - Introduce students to the modern computational functional calculus based on rational function approximation theory.

3. **Timetable**

| Date & Time | Content |
|---|---|
| **Virtual phase** | |
| **14.06.2024 2 hrs exact time will be confirmed later** | **Lecture 1: Review of Course Material**<br>We review linear algebra, complex and real analysis tools necesary for the study of spectral theory. |
| **20.06.2024 2 hrs exact time will be confirmed later** | **Lecture 2: Introduction to the Topic**<br>Examples of linear operators on Hilbert spaces and their applications in mathematical physics and control theory. |
| **Physical phase at the Inter University Centre, Dubrovnik** | |
| **26.06.2024 11:00-13:00 CET** | **Lecture I:**<br>Closed operators and their spectrum. Operators defined as sesquilinear forms. |
| **27.06.2024 11:00-13:00 CET** | **Lecture II:**<br>Spectral projections, resolvent formulae and Dunford integral |
| **27.06.2024 15:00-17:00 CET** | **Lecture III:**<br>Integration with respect to the spectral measure |
| **28.06.2024 9:00-11:00 CET** | **Lecture IV:**<br>Sturm-Liouville Operators and applications |
| **29.06.2024 9:00-11:00 CET** | **Lecture V:**<br>Resolvent-based approach to evaluate the spectral measure, regularization using rational convolution methods. |

**Recommended reading/textbook list for the course:**

- E. Brian Davies: Linear Operators and their Spectra, Cambridge Studies in Advanced Mathematics
- J. Weidmann: Linear Operators in Hilbert Spaces, Springer
- K. Schmudgen: Unbounded Self-adjoint Operators on Hilbert Space
- M. Colbrook, A. Horning, A. Townsend, "Computing spectral measures of self-adjoint operators," SIAM Review, 63(3), 489-524, 2021

4. **Mode of delivery:**

- Students need to read the materials and complete the problem assignments.
- Virtual Seminars via Zoom or Microsoft Teams.
- Face-to-face classes at the Inter University Centre, Dubrovnik.
- Virtual and face to face group meetings

5. **Period**: June 2024

   Virtual phase: 14.06 and 20.06.2024
   Physical phase: 24.06 - 30.06.2024

6. **Students:** Open to students from Mathematics Bachelor and Master

7. **Workload**:

   - Independent work (Reading materials, homework problems) 14 h
   - Lectures 14 h (Virtual 4 h, Face to face 10 h)
   - Group project 14 h
   - Total number of hours: 42 h

8. **Assessment methods:**

   In order to pass this course, the following assessment criteria will be used:
   - Compulsory participation in the virtual and physical lectures
   - Compulsory reading and homework assignments
   - Final group project assignment, due before August 31, 2024.

**Grading:** Pass/Fail or Satisfactory/Unsatisfactory

9. **Teaching language:** English

10. **ECTS:** 5 in total together with the other two topics:
    - Spectral Theory
    - Elements of the Theory of Hilbert Spaces,

    in the Summer School. Total workload of the three topics is 126 h.

**Topic: Algebraic Geometry**
**Instructor: Prof. Dr. Anca Mustata, University College Cork**

1. **Course description**: Algebraic Geometry is an area of mathematics whose basic objects of study are solution sets of polynomial equations in multiple variables – these are the building blocks of algebraic varieties. In this course we will introduce and work with algebraic methods designed to describe geometric properties of algebraic varieties, with a focus on the study of algebraic curves.

2. **Course goals:**
   - To introduce the main concepts and techniques used in the study of solution sets of polynomial equations.
   - To provide the students with concrete examples of algebraic varieties and their geometric feature, which can serve as a solid basis for a graduate course in Algebraic Geometry.

3. **Timetable**

| Date & Time | Content |
|---|---|
| **Virtual phase** | |
| **14.06.2024 2 hrs exact time will be confirmed later** | **Lecture 1: Review of Course Material**<br>We review linear algebra methods used in the study of solution sets of polynomial equations: elimination methods, resultant of two polynomials, secant and tangent methods for finding rational points on curves. |
| **20.06.2024 2 hrs exact time will be confirmed later** | **Lecture 2: Introduction to the Topic**<br>We introduce the notions of affine and projective varieties, and compare algebraic curves over the fields of rational, real and complex numbers. |
| **Physical phase at the Inter University Centre, Dubrovnik** | |
| **24.06.2024 9:00-11:00 CET** | **Lecture I:**<br>**A**lgebraic curves: tangent cone, smooth and singular points, conics and cubics. |
| **25.06.2024 9:00-11:00 CET** | **Lecture II:**<br>Bezouts' formula for the intersection of plane projective curves. The group structure on an elliptic curve. |
| **27.06.2024 9:00-11:00 CET** | **Lecture III:**<br>The Euler number and genus; the Riemann-Hurwitz formula. The Genus - Degree formula for plane curves. |
| **28.06.2024 11:00-13:00 CET** | **Lecture IV:**<br>The ideal-variety correspondence. Hilbert's Nullstellensatz for affine and projective varieties. Regular maps. Veronese and Segre embeddings. |

| | |
|---|---|
| **28.06.2024**<br>**15:00-17:00 CET** | **Lecture V:**<br>Hilbert polynomials, dimension, degree and arithmetic genus of projective varieties. |

**Recommended reading/textbook list for the course:**

- Miles Reid: Undergraduate Algebraic Geometry, Cambridge University Press, 2000
- Frances Kirwan: Complex Algebraic Curves, Cambridge University Press, 1992.
- David Cox, John Little, Donal O'Shea: Ideals, Varieties and Algorithms. chapters 1, 3, 4, 5, 7, 8, 9

**4. Mode of delivery:**

- Students need to read the materials and complete the problem assignments.
- Virtual Seminars via Zoom or Microsoft Teams.
- Face-to-face classes at the Inter University Centre, Dubrovnik.
- Virtual and face to face group meetings

**5. Period**: June 2024

Virtual phase: 14.06 and 20.06.2024
Physical phase: 24.06 - 30.06.2024

**6. Students:** Open to students from Mathematics Bachelor and Master

**7. Workload**:

- Independent work (Reading materials, homework problems) 14 h
- Lectures 14 h (Virtual 4 h, Face to face 10 h)
- Group project 14 h
- Total number of hours: 42 h

**8. Assessment methods:**

In order to pass this course, the following assessment criteria will be used:

- Compulsory participation in the virtual and physical lectures
- Compulsory reading and homework assignments
- Final group project assignment, due before August 31, 2024.

**Grading:** Pass/Fail or Satisfactory/Unsatisfactory

**9. Teaching language:** English

**10. ECTS:** 5 in total together with the other two topics:

- Spectral Theory
- Elements of the Theory of Hilbert Spaces,

in the Summer School. Total workload of the three topics is 126 h.

**Topic: Elements of the theory of Hilbert spaces**
**Instructor: Prof. Dr. Andrzej Łuczak, Łodz University**

1. **Course description**: The aim of the course is to present some fundamental topics of the theory of Hilbert spaces such as linear forms and operators, adjoint and selfadjoint operators, orthogonal projections and basic topologies on the space B(H).

2. **Course goals:**
   - To provide the students with basic knowledge of Hilbert space theory allowing them to study more advanced topics such as e.g. unbounded operators or spectral theory of linear operators on Hilbert spaces.

3. **Timetable**

| Date & Time | Content |
|---|---|
| **Virtual phase** | |
| **14.06.2024 2 hrs exact time will be confirmed later** | **Lecture 1: Review of Course Material** Presentation of the subject of the course and its connection with other fields of mathematics. Overview of the students' mathematical background necessary for the understanding of the lectured material. |
| **20.06.2024 2 hrs exact time will be confirmed later** | **Lecture 2: Introduction to the Topic** Brief recalling of basic notions of normed (Banach) spaces, bounded linear operators and functionals, with some typical examples in finite and infinite dimensions. |
| **Physical phase at the Inter University Centre, Dubrovnik** | |
| **24.06.2024 11:00-13:00 CET** | **Lecture I: Inner products on linear spaces. Schwarz inequality. Polarization formula. Definition and examples of Hilbert spaces.** |
| **24.06.2024 15:00-17:00 CET** | **Lecture II: Orthogonality. Orthogonal bases. Bessel inequality and Parseval identity. Separable Hilbert spaces.** |
| **25.06.2024 11:00-13:00 CET** | **Lecture III: The weak topology and the notion of the dual space. Riesz's representation theorem.** |
| **25.06.2024 15:00-17:00 CET** | **Lecture IV: Bounded linear forms and bounded linear operators. The adjoint operator. Unitary operators. Strong-operator and weak-operator topologies.** |
| **26.06.2024 9:00-11:00 CET** | **Lecture V: The lattice of projections. Union (∨) and intersection (∧) of projections. Properties of projections.** |

**Recommended reading/textbook list for the course:**

1. **R.V. Kadison and J.R. Ringrose "Fundamentals of the Theory of Operator Algebras I" Chapter 2, Academic Press, New York, 1983.**
2. **G.K. Pedersen "Analysis Now" Chapter 3, Springer, New York, 1989.**
3. **W. Rudin "Functional Analysis" Chapter 12, McGraw-Hill, New York, 1973.**

4. **Mode of delivery:**

   - Students need to read the materials and complete the problem assignments.
   - Virtual Seminars via Zoom or Microsoft Teams.
   - Face-to-face classes at the Inter University Centre, Dubrovnik.
   - Virtual and face to face group meetings

5. **Period**: June 2024

   Virtual phase: 14.06 and 20.06.2024
   Physical phase: 24.06 - 30.06.2024

6. **Students:** Open to undergraduate students who have received at least 90 ECTS credits in mathematical courses or for 1st year graduate students.

7. **Workload**:

   - Independent work (Reading materials, homework problems) 14 h
   - Lectures 14 h (Virtual 4 h, Face to face 10 h)
   - Group project 14 h
   - Total number of hours: 42 h

8. **Assessment methods:**

   In order to pass this course, the following assessment criteria will be used:
   - Compulsory participation in the virtual and physical lectures
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   - Final group project assignment, due before August 31, 2024.

**Grading:** Pass/Fail or Satisfactory/Unsatisfactory

9. **Teaching language:** English

10. **ECTS:** 5 in total together with the other two topics:
    - Spectral Theory
    - Algebraic Geometry,

    in the Summer School. Total workload of the three topics is 126 h.

# Programme Overview for the week 24-30 June 2024

| Time Slot | Monday June 24 | Tuesday June 25 | Wednesday June 26 | Thursday June 27 | Friday June 28 | Saturday June 29 | Sunday June 30 |
|---|---|---|---|---|---|---|---|
| **9:00-11:00** | Algebraic Geometry I | Algebraic Geometry II | Hilbert Space Theory V | Algebraic Geometry III | Spectral Theory IV | Spectral Theory V | Discussion |
| **11:00-13:00** | Hilbert Space Theory I | Hilbert Space Theory III | Spectral Theory I | Spectral Theory II | Algebraic Geometry IV | Question and Answer | Discussion |
| **13:00-15:00** | LUNCH and Discussion | LUNCH and Discussion | LUNCH and Discussion | LUNCH and Discussion | LUNCH and Discussion | LUNCH and Discussion | LUNCH |
| **15:00-17:00** | Hilbert Space Theory II | Hilbert Space Theory IV | Free Afternoon | Spectral Theory III | Algebraic Geometry V | Free Afternoon | Free Afternoon |